Vectors & Scalers

Vectors & Scalars

SCALAR

The physical quantity which has a magnitude but no specific direction. eg. - distance, mass, speed.

VECTOR

The physical quantity which has a magnitude as well as direction and follows the vector law of addition. eg.- Force, velocity, displacement, momentum.

NOTE -1 Current is not a vector quantity though it has direction and magnitude as it does not follow vector law of addition.

REPRESENTATION OF A VECTOR QUANTITY

A vector is represented by drawing an arrow propotional in length to the physical quantity being represented.

- A vector variable is represented by an arrow over english or greek alphabet.
- The same alphabet without the vector sign represents the magnitude of the vector and the same alphabet with a cap sign represents the direction of the vector.

Practice Time

(a) $\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

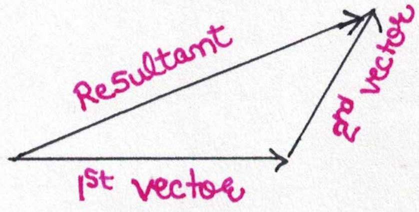
(b) $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

(c) $\tan\left(\frac{3\pi}{4}\right) = -1$

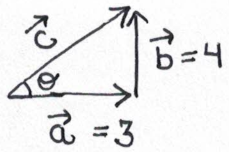
(d) $\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

TRIANGLE LAW OF ADDITION

To add two vectors, using triangle law, place the tail of the second vector on the head of the first vector. Now complete the triangle and the resultant vector is given by the arrow starting from tail of the first vector to the head of the second vector.



Eg -



$\vec{c} = \vec{a} + \vec{b}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

For example -

If $\vec{a}$ = 1 Km North

then, a = 1 km and $\hat{a}$ = North

INVERSE TRIGONOMETRIC NOTATION FOR ANGLES

$\sin^{-1}(x)$ represents an angle whose Sin is x

Eg - $\tan^{-1}(x) = \tan^{-1}(1) = 45^\circ$

$\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

RADIAN MEASURE OF AN ANGLE

$360^\circ = 2\pi$ radian

If the angle (θ) of an arc is expressed in radians then the arc length is simply

$$\boxed{l = r\theta}$$

Trigonometric ratios for angles $> 90^\circ$

$\sin(\pi - \theta) = \sin\theta$

$\cos(\pi - \theta) = -\cos\theta$

$\tan(\pi - \theta) = -\tan\theta$

$\sin(\pi + \theta) = -\sin\theta$

$\cos(\pi + \theta) = -\cos\theta$

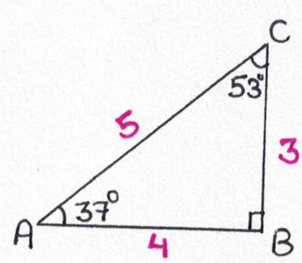
$\tan(\pi + \theta) = \tan\theta$

NOTE - 2 $\sin 37^\circ = \frac{3}{5}$

$\cos 37^\circ = \frac{4}{5}$

$\sin 53^\circ = \frac{4}{5}$

$\cos 53^\circ = \frac{3}{5}$



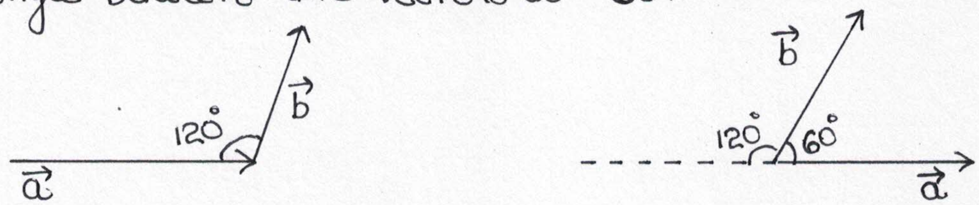
NOTE - 3 Vector sum is commutative i.e.

$\vec{a} + \vec{b} = \vec{b} + \vec{a}$

ANGLE BETWEEN TWO VECTORS

The angle between two vectors is defined as the non-reflex angle, between the vectors when they are joined tail to tail.

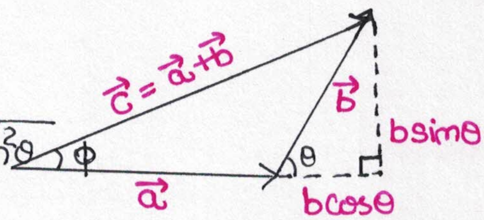
Eg - The angle between the vectors is 60°.



ANALYTICAL ADDITION OF VECTORS

When we consider two vectors $\vec{a}$ and $\vec{b}$ represented for finding the resultant $(\vec{a}+\vec{b})$ using the triangle law as shown in fig.

Using pythagoras theorem



$$|\vec{c}| = \sqrt{(a+b\cos\theta)^2 + (b\sin\theta)^2}$$

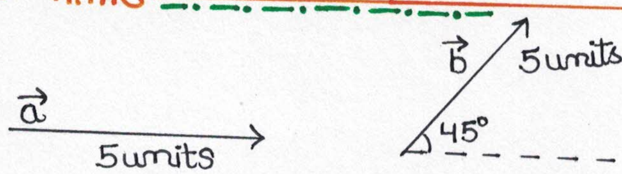
$$|\vec{c}| = \sqrt{a^2+b^2\cos^2\theta + 2ab\cos\theta + b^2\sin^2\theta}$$

$$|\vec{c}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

and direction is given by

$$\phi = \tan^{-1}\left(\frac{b\sin\theta}{a+b\cos\theta}\right)$$

Practice Time



$$\text{resultant} = \sqrt{25+25+50\times\frac{1}{\sqrt{2}}}$$

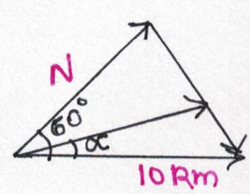
$$= \sqrt{50+\frac{50}{\sqrt{2}}}$$

$$= \sqrt{\frac{50\sqrt{2}+50}{\sqrt{2}}}$$

$$\phi = \tan^{-1}\left(\frac{5/\sqrt{2}}{5+5/\sqrt{2}}\right) = \tan^{-1}\left(\frac{5}{5\sqrt{2}+5}\right)$$

$$\phi = \tan^{-1}\left(\frac{1}{\sqrt{2}+1}\right)$$

A man walks 10 Km towards east & then another 10 km towards 60° North east. Find its resultant & direction.



$$\vec{c} = \sqrt{200+200\times\frac{1}{2}}$$

$$= \sqrt{300}$$

$$= 10\sqrt{3}$$

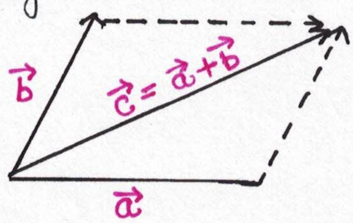
$$\alpha = \tan^{-1}\left(\frac{5\sqrt{3}}{10+15}\right)$$

$$\alpha = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\alpha = 30^\circ$$

PARALLELOGRAM LAW OF ADDITION

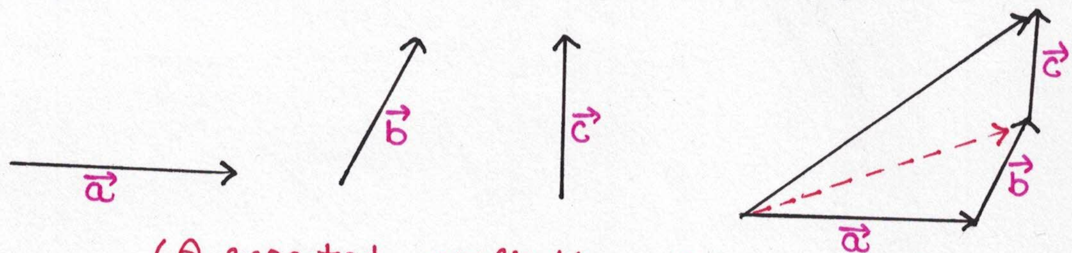
Here we place the vector's tail to tail and complete the parallelogram. Now the resultant is given by diagonal of the parallelogram passing through the common tail.



NOTE-4 For any given vector $\vec{a}$, the symbols a and $|\vec{a}|$ mean exactly the same thing.

POLYGON LAW OF ADDITION

Here we join all the vectors to be added in a head to tail configuration and now the resultant is given by the vector joining tail of the first vector to the head of the last vector.



(A repeated application of Triangle law)

NEGATIVE OF A VECTOR

Negative of a vector is a vector with the same magnitude but with reversed direction.

Eg - $\vec{a} = 1 \text{ unit } \hat{N}$

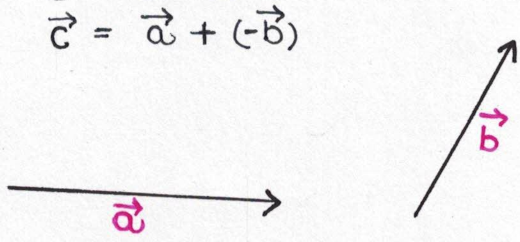
$-\vec{a} = 1 \text{ unit } \hat{S}$

SUBTRACTION OF VECTORS

Subtraction is same as the addition of the negatives.

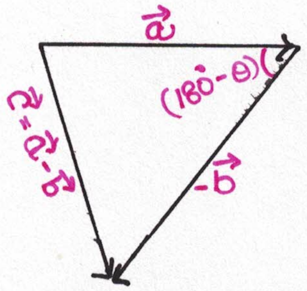
$$\vec{c} = \vec{a} - \vec{b}$$

$$\vec{c} = \vec{a} + (-\vec{b})$$



$$|\vec{c}| = \sqrt{a^2+b^2+2ab\cos(180^\circ-\theta)}$$

$$|\vec{c}| = \sqrt{a^2+b^2-2ab\cos\theta}$$



UNIT VECTOR

A vector having magnitude and some specific direction is called Unit Vector. It is unitless.

- Any vector divided by its magnitude gives the unit vector in the same direction.

For eg- If $\vec{a} = 5\text{km}\ \hat{N}$

$a = 5\text{km}$

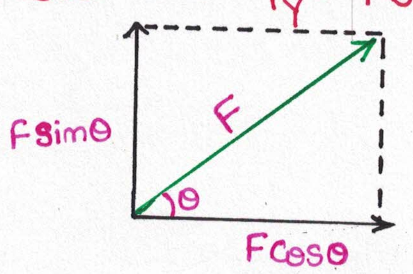
$\hat{a} = \dfrac{\vec{a}}{a} \quad 1\hat{N}$

RECTANGULAR COMPONENTS OF A VECTOR

Whenever a vector is expressed as a sum of two mutually perpendicular vectors, then we called the vectors as rectangular components of the vector.

There are infinite ways of doing this. To resolve a vector into mutually perpendicular components, imagine a rectangle whose diagonal is the given vector. The sides of such rectangle are the rectangular components.

For instance let there be a force F inclined at an angle θ with the horizontal, then its horizontal and vertical components are given by $F_x = F\cos\theta$ and $F_y = F\sin\theta$

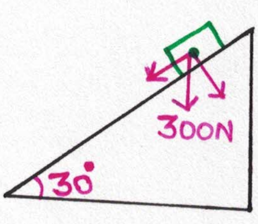


Practice Time

(1) Resolve the wt. vector of block into components

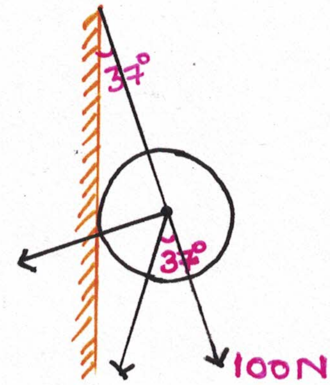
$W_{\parallel} = mg\sin\theta = 300 \times \frac{1}{2} = 150$

$W_{\perp} = mg\cos\theta = 300 \times \frac{\sqrt{3}}{2} = 150\sqrt{3}$



(2) $\parallel$ to thread $= 100 \times \frac{4}{5} = 80N$

$\perp$ to thread $= 100 \times \frac{3}{5} = 60N$

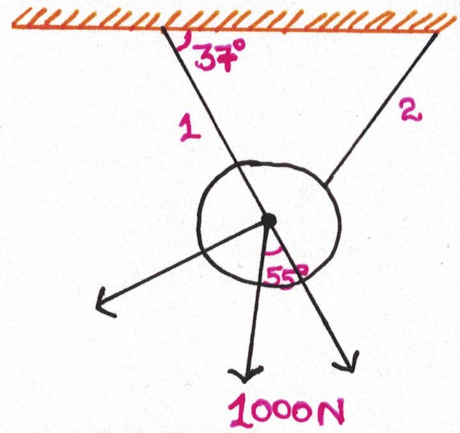


(3) $\perp = 1000 \times \frac{4}{5}$

$= 800$

$\parallel = 1000 \times \frac{3}{5}$

$= 600$

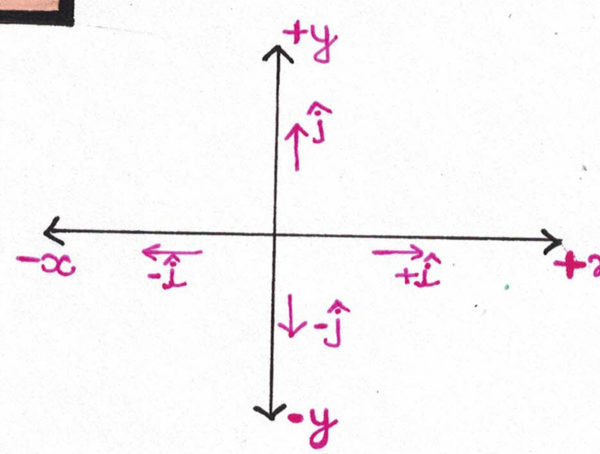


UNIT VECTORS ALONG COORDINATE AXIS

Any vector, say $\vec{a}$ drawn in the x-y plane can always be written in terms of its components $\parallel$ to the x and y-axis respectively.

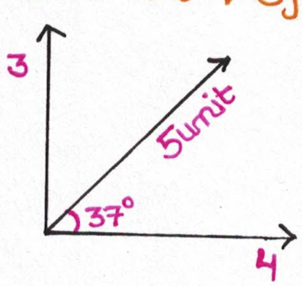
For eg - In the shown fig., $\vec{a}$ can be written as

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$



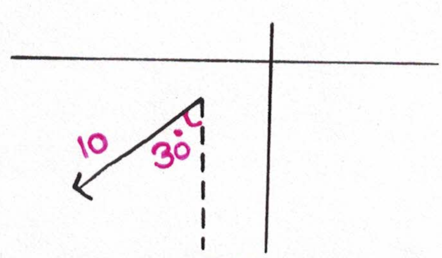
Such representation is called Cartesian representation.

(1) $\vec{5} = 4\hat{i} + 3\hat{j}$



(2) $\vec{10} = -10\cos 30\hat{i} - 10\sin 30\hat{j}$

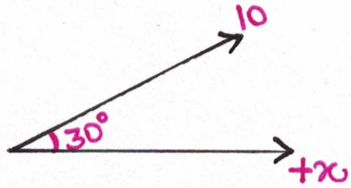
$= -5\hat{i} - 5\sqrt{3}\hat{j}$



POLAR REPRESENTATION OF VECTORS

When a vector is specified in terms of its magnitude and angle which it makes with the x-axis, we call it polar representation.

For eg. - $10 @ 30^\circ$



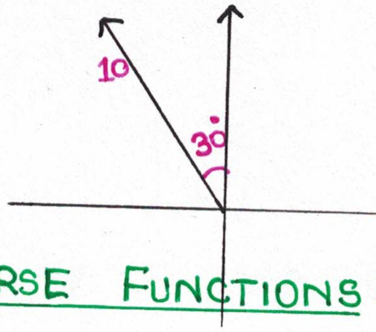
CONVERTING FROM POLAR FORM TO CARTESIAN FORM

Consider a vector $\vec{R}$ having a magnitude $\vec{R}$ making an angle θ with the x-axis as shown.

$$\vec{r} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

■ Represent the shown vector in cartesian form:

$$\vec{v} = -5\hat{i} + 5\sqrt{3}\hat{j}$$
$$= 10\cos 120\,\hat{i} + 10\sin 120\,\hat{j}$$



PRINCIPAL RANGES FOR INVERSE FUNCTIONS

$\sin^{-1} = -90^\circ$ to 90°

$\tan^{-1} = -90^\circ$ to 90°

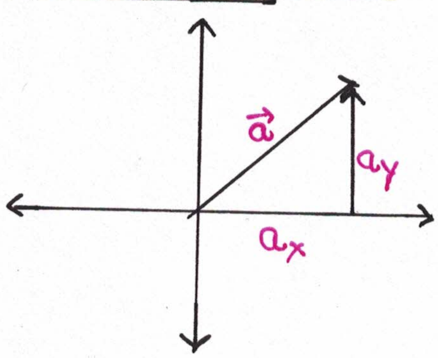
$\cos^{-1} = 0^\circ$ to 180°

CONVERTING FROM CARTESIAN FORM TO POLAR FORM

Consider a vector $\vec{a}$, as represented in cartesian form $\vec{a} = a_x\hat{i} + a_y\hat{j}$ as shown in fig.

$$a = \sqrt{a_x^2 + a_y^2}$$

$$\tan\theta = \frac{a_y}{a_x}$$



$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) \text{ if } a_x \text{ is positive}$$

$$\theta = 180^\circ + \tan^{-1}\left(\frac{a_y}{a_x}\right) \text{ if } a_x \text{ is negative}$$

Convert the following from cartesian to Polar forms:

(a) $3\hat{i} + 4\hat{j} = 5 @ 53^\circ$

(b) $-4\hat{i} + 3\hat{j} = 5 @ 143^\circ$

(c) $-3\hat{i} - 4\hat{j} = 5 @ 233^\circ$

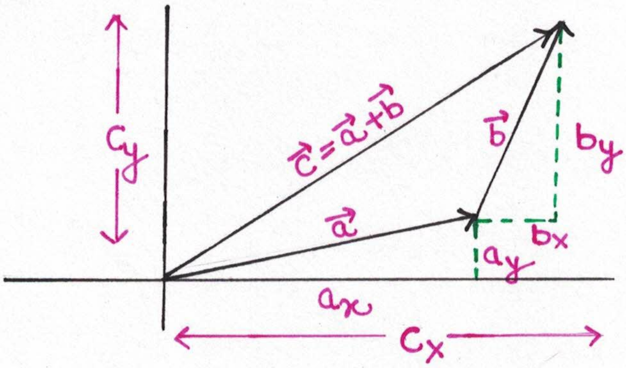
(d) $4\hat{i} - 3\hat{j} = 5 @ \tan^{-1}\left(-\frac{5}{4}\right) = 5 @ -37^\circ$

ADDITION OF VECTORS IN CARTESIAN FORM

Let $\vec{a}$ and $\vec{b}$ be two vectors given by

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$
$$\vec{b} = b_x\hat{i} + b_y\hat{j}$$

represented for addition by triangle law in cartesian plane as shown



Let c be the resultant given by ($\vec{c} = c_x\hat{i} + c_y\hat{j}$) then it is evident from the figure that

$$c_x = a_x + b_x$$
$$c_y = a_y + b_y$$

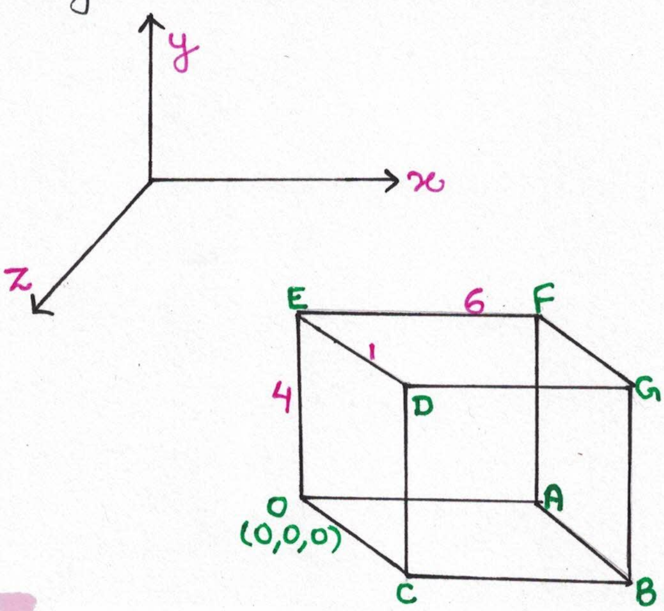
For eg→ $\vec{a} = 3\hat{i} - 4\hat{j}$

$\vec{b} = 4\hat{i} + 8\hat{j}$

$\vec{a} + \vec{b} = 7\hat{i} + 4\hat{j}$

THE VECTOR $\hat{k}$

To represent the location of a point in space, we need a third coordinate called the z-coordinate which represents the height or depth over the x-y plane which we need to travel to reach that desired point. This is called the z-direction. The unit vector along this direction is $\hat{k}$.



A = 6, 0, 0
B = 6, 0, 1
C = 0, 0, 1
D = 0, 4, 1
E = 0, 4, 0
F = 6, 4, 0
G = 6, 4, 1

POSITION VECTOR

The position vector of a point in space is defined as the vector joining the origin to that point.

For eg: $\vec{r}_G = 6\hat{i} + 4\hat{j} + \hat{k}$

MAGNITUDE OF A VECTOR IN 3 DIMENSION

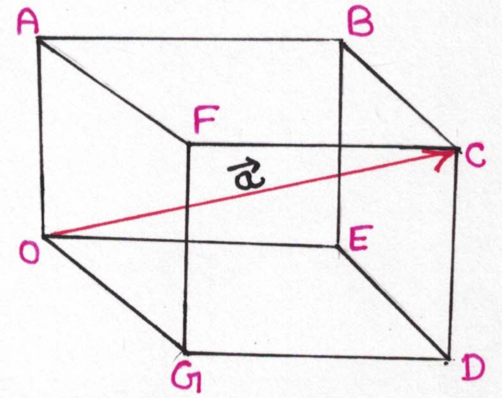
$OC^2 = OB^2 + BC^2$

$OB^2 = OA^2 + AB^2$

$OC^2 = OA^2 + AB^2 + BC^2$

$a^2 = a_x^2 + a_y^2 + a_z^2$

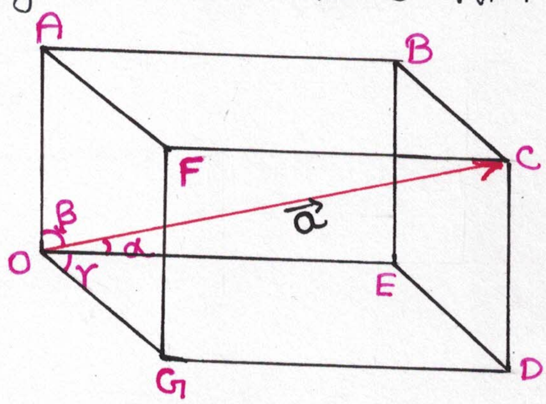
$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$



DIRECTION COSINE OF A VECTOR

The cosines of the angles that a given vector makes with x, y and z axis respectively.

Cos α, Cos β and Cos γ are called direction cosines.



ΔCEO has

$\angle CEO = 90°$

$\cos\alpha = \dfrac{OF}{OC}$

$= \dfrac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}} = \dfrac{a_x}{a}$

$\cos\beta = \dfrac{a_y}{a}$

$\cos\gamma = \dfrac{a_z}{a}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

MULTIPLICATION OF A VECTOR BY SCALAR

Whenever a vector is multiplied by a scalar, the magnitude gets multiplied and the direction remains same if it is a positive scalar and direction reverses if scalar is negative.

Eg: $\vec{F} = 6N$ east

$t = 3$ sec

$\vec{F}.t = 18$ N sec. east

$= -18$ N sec. west

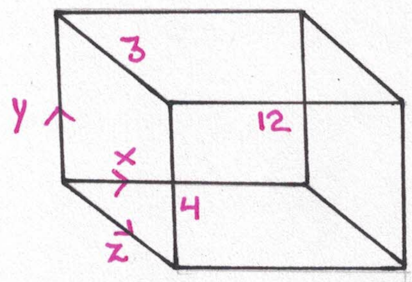
DISPLACEMENT VECTOR

If a particle moves from one position to another then the change in its position vector is called displacement vector.

$$\vec{S} = \vec{r}_f - \vec{r}_i$$

$\vec{r}_f$ = final position vector

$\vec{r}_i$ = initial position vector



Eg: $S = 12\hat{i} - 4\hat{j} - 3\hat{k}$

$\vec{r}_i = 4\hat{j} + 3\hat{k}$

$\vec{r}_f = 12\hat{i}$

$\vec{S} = \vec{r}_f - \vec{r}_i$

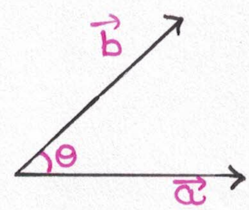
$= 12\hat{i} - 4\hat{j} - 3\hat{k}$

$|\vec{S}| = \sqrt{12^2 + 4^2 + 3^2} = 13m$

DOT PRODUCT (SCALAR PRODUCT)

Let there be two vectors $\vec{a}$ and $\vec{b}$ having an angle θ between them. Then we define their product as

$$\boxed{d = ab\cos\theta}$$



For eg: $\vec{a}.\vec{b} = 12\cos 37^\circ$

$= \frac{48}{5}$

DOT PRODUCT IN CARTESIAN FORM

$\hat{i}.\hat{i} = 1$

$\hat{i}.\hat{j} = 0$

$\hat{j}.\hat{j} = 1$

$\hat{j}.\hat{k} = 0$

$\hat{k}.\hat{k} = 1$

$\hat{k}.\hat{i} = 0$

NOTE : Dot product is distributive i.e.

$$\vec{a}.(\vec{b}+\vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$$

Dot product is commutative i.e.

$$\vec{a}.\vec{b} = \vec{b}.\vec{a}$$

Suppose now we have two have two vectors $\vec{a}$ & $\vec{b}$ given by $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$, then

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$$

ANGLE BETWEEN TWO VECTORS

We know that $\vec{a}.\vec{b} = ab\cos\theta$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{ab}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{\vec{a}.\vec{b}}{ab}\right)}$$

PRACTICE TIME

1.) $\vec{a} = 5\sqrt{3}\hat{i} + 5\hat{j}$

$\vec{b} = -6\hat{j}$

$$\frac{\vec{a}.\vec{b}}{ab} = \cos\theta$$

$$\frac{-\cancel{30}}{-\cancel{10}\times\cancel{6}\,2} = \cos\theta$$

$$\theta = 120^\circ$$

2.) $\vec{a} = \hat{i}+\hat{j}+\hat{k}$

$\vec{b} = -\hat{i}-\hat{j}+\sqrt{2}\hat{k}$

$$\cos\theta = \frac{-1-1+\sqrt{2}}{\sqrt{3}\times 2} = \frac{-2+\sqrt{2}}{2\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}-2}{2\sqrt{3}}\right)$$

ZERO VECTOR

A vector whose magnitude is 0 and direction is arbitrary.

CONDITION FOR ORTHOGONALITY (PERPENDICULARITY)

(a) Two vectors are perpendicular if their dot product is zero.

(b) If the dot product of two non-zero vectors is 0 then they are orthogonal.

PRACTICE QUESTION →

$$\left.\begin{aligned}\vec{a} &= 3\hat{i} - 2\hat{j}\\ \vec{b} &= p\hat{i} + q\hat{j}\end{aligned}\right\}$$ They are $\perp$ & $|b| = \sqrt{13}$

$$3p - 2q = 0$$

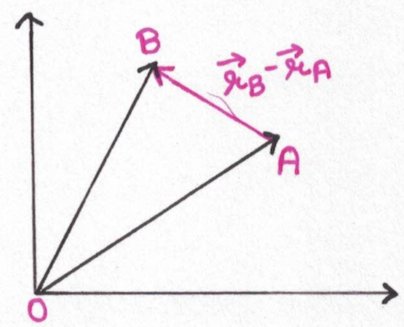
$$3p = 2q$$

$$\sqrt{p^2 + \frac{9}{4}q^2} = 13$$

$$\frac{\cancel{13}}{4}p^2 = \cancel{13}$$

$$\boxed{\begin{array}{c} p = \pm 2 \\ q = \pm 3 \end{array}}$$

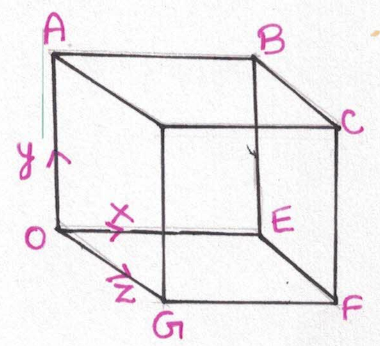
RELATIVE POSITION VECTOR



Let there be two points in space, A and B whose position vectors are given by $\vec{r}_A$ and $\vec{r}_B$ then the position of B as seen from A is defined as vector joining A to B which is also called position of B relative to A (r_{BA}). As it is clear from the figure position of B relative to A, is given by

$$\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$$

PRACTICE TIME



1) What is $\vec{r}_G$, $\vec{r}_C$, $\vec{r}_A$, $\vec{r}_{CG}$ and $\vec{r}_{AG}$
Find angle b/w $\vec{GA}$ and $\vec{GC}$.

$$\vec{r}_G = 2\hat{k}$$
$$\vec{r}_C = 2\hat{i} + 2\hat{j} + 2\hat{k}$$
$$\vec{r}_A = 2\hat{j}$$
$$\vec{r}_{CG} = 2\hat{i} + 2\hat{j}$$
$$\vec{r}_{AG} = 2\hat{j} - 2\hat{k}$$

angle between GA & GC $= \dfrac{\cancel{4}}{\cancel{2}\sqrt{2} \times \cancel{2}\sqrt{2}}$

$$\cos\theta = \frac{1}{2}$$

$$\boxed{\theta = 60^\circ}$$

2) Find angle b/w body diagonals of cube.

$$\vec{OC} = 2\hat{i} + 2\hat{j} + 2\hat{k}$$
$$\vec{GB} = 2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{OC} \cdot \vec{GB} = \frac{\cancel{4} + \cancel{4} - \cancel{4}}{\cancel{2}\sqrt{3} \times \cancel{2}\sqrt{3}} = \frac{1}{3}$$

NOTE :- $\vec{r}_{AG} = \vec{GA}$ (Starting from G to A)

PROJECTION OF A VECTOR ON ANOTHER VECTOR (COMPONENT)

Let there be two vectors $\vec{a}$ and $\vec{b}$ represented by lengths OA and OB, then the projection of A on B is obtained by joining the two vectors from tail to tail and dropping a perpendicular from head of A on vector B (say pt. A'). Now OA' is the projection. As it is clear from the figure,

$$\text{projection of } \vec{a} \text{ on } \vec{b} = a\cos\theta$$
$$= \cancel{a}\left(\frac{\vec{a}.\vec{b}}{\cancel{a}b}\right)$$
$$= \frac{\vec{a}.\vec{b}}{b}$$
$$= \vec{a}.\frac{\vec{b}}{b}$$
$$= \vec{a}.\hat{b}$$

Similarly, projection of $\vec{b}$ on $\vec{a}$ = $\vec{b}.\hat{a}$

CROSS PRODUCT OF VECTORS

Let there be two vectors $\vec{a}$ and $\vec{b}$, having an angle θ between them, then their cross product $\vec{c}$ is given by

$$\vec{c} = \vec{a} \times \vec{b}$$

and magnitude is given by

$$C = ab\sin\theta$$

and direction is given by right hand thumb rule or the right hand screw rule.

RIGHT HAND SCREW RULE

Join the vectors to be crossed tail to tail and rotate a screw at the junction of the common tails from first vector to the second vector through the non-reflex angle. The direction of the rotation of the screw is the direction of cross product.

RIGHT HAND THUMB RULE

Place the palm of the right hand perpendicular to the palm of the vectors such that the little finger is along the first vector and the curl of the fingers points towards the other vector through the non-reflex angle. Now the thumb gives the direction of the cross product.

(Base of the palm / wrist should be at the common tail.)

CROSS PRODUCT IN CARTESIAN FORM

$\hat{i} \times \hat{i} = 0$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{j} = 0$

$\hat{j} \times \hat{k} = \hat{i}$

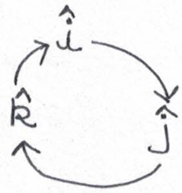
$\hat{k} \times \hat{k} = 0$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{i} \times \hat{k} = -\hat{j}$



NOTE

1.) Cross product is not commutative

$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$

2.) Cross product is distributive i.e.

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

MATRIX METHOD FOR CROSS PRODUCT

Let there be two vectors given by

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

by multiplying out, we get this

$$\vec{a} \times \vec{b} = \hat{i}(a_y b_z - a_z b_y) + \hat{j}(a_z b_x - a_x b_z) + \hat{k}(a_x b_y - a_y b_x)$$

To do the same process in an easy manner, we represent the data as shown in the following matrix and now the cross product is given by the determinant of this matrix.

$$\begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

NOTE

1.) If two vectors are parallel, then this cross product will be zero.

$$\frac{a_x}{b_x} = \frac{a_y}{b_y} = \frac{a_z}{b_z}$$

2) Antiparallel vectors are $\rightleftarrows$ with equal magnitude and opposite direction.

LAMI'S THEOREM

CONDITION FOR EQUILIBRIUM:

A body is said to be in equilibrium if the vector sum of all the forces acting on it is 0.

SINE LAW: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

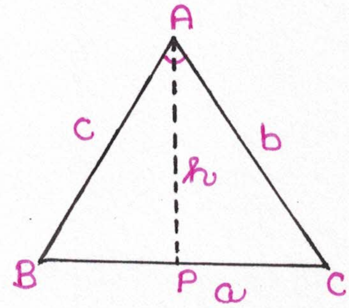
Proof:

$AP = AB \sin B$

$AP = AC \sin C$

$c \sin B = b \sin C$

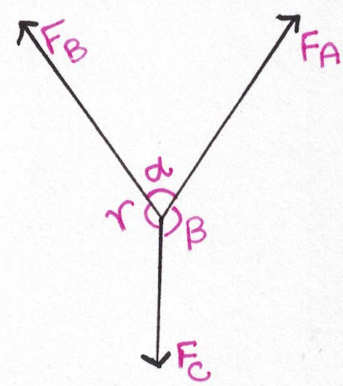
$\frac{c}{\sin C} = \frac{b}{\sin B}$



STATEMENT OF LAMI'S THEOREM

Let a particle or a body be in equilibrium under the influence of three forces F_A, F_B and F_C, arrange tail to tail then

$$\frac{F_A}{\sin\alpha} = \frac{F_B}{\sin\beta} = \frac{F_C}{\sin\gamma}$$



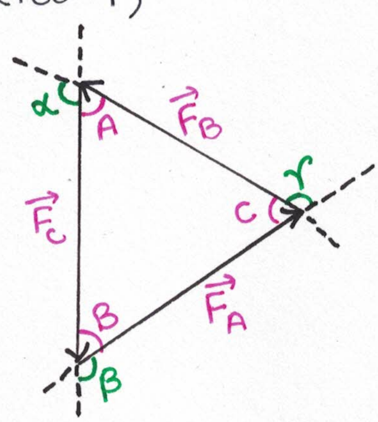
PROOF

As resultant force is 0, Closed triangle will be formed

$$\frac{F_A}{\sin A} = \frac{F_B}{\sin B} = \frac{F_C}{\sin C}$$

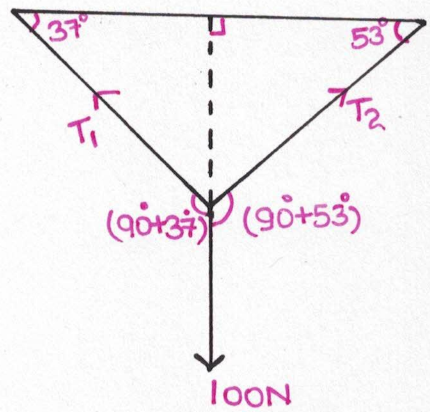
$$\frac{F_A}{\sin(180^\circ-\alpha)} = \frac{F_B}{\sin(180^\circ-\beta)} = \frac{F_C}{\sin(180^\circ-\gamma)}$$

$$\frac{F_A}{\sin\alpha} = \frac{F_B}{\sin\beta} = \frac{F_C}{\sin\gamma}$$



PRACTICE TIME

$$\frac{T_1}{\sin 37^\circ} = \frac{T_2}{\sin 53^\circ} = 100$$



$$\frac{T_1 \times 5}{3} = 100$$

$$T_1 = 20 \times 3$$

$$\boxed{T_1 = 60}$$

$$\frac{T_2}{4} \times 5 = 100$$

$$\boxed{T_2 = 80}$$